Measuring the Wavelength of a Laser with a Ruler

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This article reviews how the wavelength of a laser was measured using a metal ruler. The apparatus used a ruler to reflect a laser beam onto a wall. At small angles between the ruler’s surface and the laser a diffraction pattern was observed on the wall. Due to the dark millimeter notches essentially being non-reflective, a model was created for the system that calculated a difference in optical path length to describe the diffraction pattern. The diffraction pattern was dependent on wavelength, so by taking the necessary measurements of length in the apparatus the angles at which fringes could be found could be determined. The wavelength was measured with a five percent error.

I. INTRODUCTION

When light is passed through a hole and onto a screen a spot can easily be seen. As the size of the hole decreases, the spot on the screen respectfully decreases; however, when the hole becomes very small the spot on the screen starts getting larger and the light begins to spread out. This phenomena is known as diffraction and is a direct result of the finiteness and scale of the wavelength of light as it interacts with the hole.

Diffraction is commonly used in a wide variety of sciences because the diffraction patterns created carry a great deal of information about the diffracting element; the hole for instance in the example. The patterns can also carry much information about repetitive arrays of diffracting elements, known as diffraction gratings. This is particularly useful because nature tends to have periodicity in structures, a good example is crystalline solids. One can think of using diffraction techniques as a way to use optics of known wavelengths as a tool, much like a ruler, for investigating objects of dramatically different sizes. This analysis has become increasingly important as modern technology has allowed us to control optics in a wide spectrum of wavelengths.

In this lab we looked at diffraction analysis at a different angle than previously mentioned. The task at hand was to measure the wavelength of a laser diode using only a metal ruler, a tape measure, and a wooden meter stick. While these seemed to be inadequate instruments for measuring properties of light we found them to be sufficient. Due to the material of the ruler the surface is rather reflective and because there is periodicity in the non-reflective black notches on the ruler we can use this as a type of diffraction grating, known as a reflection grating.

The laser was adjusted to glance the surface of the ruler so that a reflected diffraction pattern could be projected on a wall. The bright fringes seen were a result of constructive interference from the various points of reflection off the ruler. This constructive interference only occurred when the peak of one wave was in phase with the peak of another wave. Physically, this condition meant the difference in optical path length had to be a whole number of wavelengths in order for constructive interference to have occurred. Mathematically, the condition is represented as:

$$\Delta OPL = m\lambda$$  (1)

Where m is an integer number. The optical path length difference was modeled and derived through the geometry of the apparatus. From simple length measurements of the apparatus and the diffraction pattern, the wavelength was calculated.

II. EXPERIMENTAL SETUP

In our setup we analyzed the ruler at the point of reflection to determine the difference in optical path length through the geometry.
Using the geometry in Figure 2, we can elaborate on equation 1:

\[ m\lambda = \Delta OPL \]
\[ m\lambda = OPL_{path2} - OPL_{path1} \]
\[ m\lambda = d\cos \alpha - d\cos \beta \]

The diffraction equation to describe our apparatus then becomes:

\[ m\lambda = d(\cos \alpha - \cos \beta_m) \] (2)

### III. EXPERIMENTAL RESULTS

We set up our apparatus and took the following measurements:

Measurements with tape measure:
\[ \overline{AB} = 297.1\, \text{cm} \pm 0.5\, \text{cm} \]
\[ = 297.0\, \text{cm} \pm 0.5\, \text{cm} \]
\[ = 297.1\, \text{cm} \pm 0.5\, \text{cm} \]
\[ = 297.0\, \text{cm} \pm 0.5\, \text{cm} \]
\[ \text{Average} = 297.05\, \text{cm} \pm 0.5\, \text{cm} \]

\[ \overline{AS_0} = 298.2\, \text{cm} \pm 0.5\, \text{cm} \]
\[ = 298.2\, \text{cm} \pm 0.5\, \text{cm} \]
\[ = 298.0\, \text{cm} \pm 0.5\, \text{cm} \]
\[ = 298.1\, \text{cm} \pm 0.5\, \text{cm} \]
\[ \text{Average} = 298.125\, \text{cm} \pm 0.5\, \text{cm} \]

Measurements with meter stick:
\[ \overline{BS_0} = 48.4\, \text{cm} \pm 0.1\, \text{cm} \]
\[ = 48.4\, \text{cm} \pm 0.1\, \text{cm} \]
\[ = 48.5\, \text{cm} \pm 0.1\, \text{cm} \]
\[ = 48.2\, \text{cm} \pm 0.1\, \text{cm} \]
\[ \text{Average} = 48.375\, \text{cm} \pm 0.1\, \text{cm} \]

Measurements with ruler:
\[ S_0S_1 = 2.20\, \text{cm} \pm 0.05\, \text{cm} \]
\[ S_1S_2 = 2.10\, \text{cm} \pm 0.05\, \text{cm} \]
\[ S_2S_3 = 2.00\, \text{cm} \pm 0.05\, \text{cm} \]
\[ S_3S_4 = 2.00\, \text{cm} \pm 0.05\, \text{cm} \]

### IV. DATA ANALYSIS

Looking at equation 2, calculation of the wavelength of the laser depends on both \( \alpha \) and \( \beta_m \). We obtained \( \alpha \) by using our measurements and the Law of Cosines:

\[ C^2 = A^2 + B^2 - 2AB\cos(\angle AB) \] (3)

Looking at the triangle formed by points A, B and S0 (\( \triangle ABS_0 \)). The angle between \( \overline{AB} \) and \( \overline{AS_0} \) is equal to \( 2\alpha \):

\[ \alpha = \frac{1}{2} \cos^{-1} \left( \frac{(BS_0)^2 - (AB)^2 - (AS_0)^2}{-2(AB)(AS_0)} \right) \]
\[ \alpha \approx 4.6609^\circ \]

\( \beta \) was found the same way with the assumption that the ruler and wall were perpendicular. Each fringe of order ‘m’ has a specific \( \beta_m \) associated with it, which was used to find an individual value for \( \lambda \):

\( \beta_m \) is found using \( \triangle AS_m_0 \)

\[ \beta_m = \tan^{-1} \left( \frac{(S_n_0)}{(A0)} \right) \]
\[ (A0) = \frac{(S_n_0)}{\tan \alpha} \approx 296.676\, \text{cm} \]

Using our data we find:
\[ \beta_1 \approx 5.08273^\circ \]
\[ \beta_2 \approx 5.48485^\circ \]
\[ \beta_3 \approx 5.86733^\circ \]
\[ \beta_4 \approx 6.24927^\circ \]

Using equation 2 to solve for \( \lambda \):
\[ \lambda_1 = 625\, \text{nm} \]
\[ \lambda_2 = 636\, \text{nm} \]
\[ \lambda_3 = 644\, \text{nm} \]
\[ \lambda_4 = 659\, \text{nm} \]
Average $\lambda$: 641 nm
Standard Deviation$^1$: 12.4 nm

V. CONCLUSIONS

After the experiment the laser diode was taken to an optics lab and hooked up to an Ocean Optics USB 2000+ Spectrometer.$^2$ Our laser was found to have a wavelength of 675.42 nm which makes our experimental result about 5% off of the expected value.

While this result seems satisfactory there were some possible errors that may have worked in our favor or possibly against us which must be addressed. One of the biggest sources of error came from the orientation of the ruler. Our calculations were assuming the ruler was perfectly perpendicular to the wall. It seems from the data that our ruler was actually off a little bit which shifted the results slightly. Another source of error was the tape measure which had a high uncertainty due to one side being roughly held in place while the other side was subject to the skill of the user in removing all the slack in the tape while taking readings. While we took multiple readings for each measurement using the tape measure there still could be some error present in our measurements.

One approach we initially used was to take all the appropriate distance measurements from the reflection point to the fringe location and solve for $\beta_m$ using the law of cosines to compensate for the wall and ruler not being perfectly perpendicular; however, with our limited resources using this method had us rely on the tape measure for such measurements, which lead to precision problems in our attempts.

We found the best method is to ensure the ruler is level and perpendicular to the wall/screen (preferably by the use of an optics table if available) and use the same procedure as documented here. Multiple readings with the tape measure will reduce the uncertainty.

VI. REFERENCES


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$^1$ Calulated: $\delta = \sqrt{\sum (x_i - \bar{x})^2}$

$^2$ Spectrometer work and results by Alex Skeffington